

Aharonov-Bohm effect and persistent currents in quantum rings

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Outline

- 1 Introduction
- 2 Free electron gas ring
- 3 Tight-binding ring
- 4 Interacting spinless fermions
- 5 Summary

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Aharonov-Bohm effect

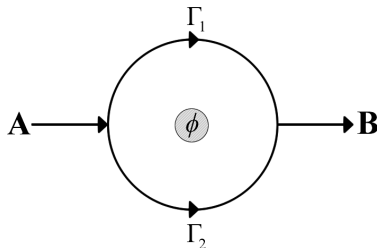


Figure: Aharonov-Bohm effect general geometry.

Aharonov-Bohm effect phase shift

$$\Delta\varphi = \frac{q}{\hbar} \oint_C \vec{A} \cdot d\vec{l} = 2\pi \frac{\phi}{\phi_o} = \phi' \quad (1)$$

Aharonov-Bohm effect

- It is a quantum mechanical topological effect
- Produces a phase shift in the electron's wavefunction
- The electron's properties are affected even if they don't "feel" a force
- Leads to persistent currents
- Leads to periodic thermodynamical properties

Persistent currents

- Currents that exist in quantum rings even with no potential difference
- Are due to Aharonov-Bohm's effect
- Have been measured experimentally, using SQUIDs, agreeing with theory

One level current

$$I_n = -\frac{\partial E_n}{\partial \phi} = bM_n \quad (2)$$

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Description and energy

- Independent electrons
- Electron's don't feel a local potential from the ions

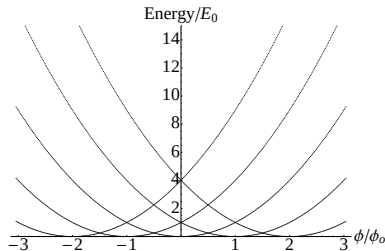
Free electron gas Hamiltonian

$$H = \frac{(\vec{p} - q\vec{A})^2}{2m} \quad (3)$$

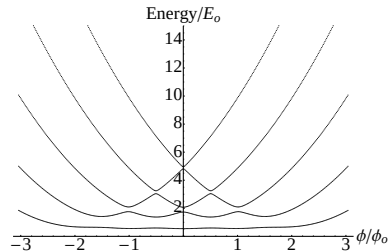
Energy

$$E_n = E_0 \left(n + \frac{\phi}{\phi_0} \right)^2, n = 0, \pm 1, \pm 2, \dots \quad (4)$$

Energy



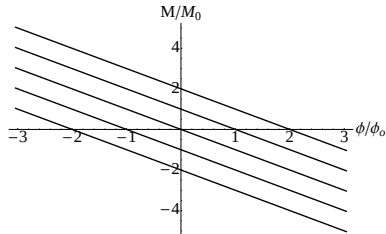
(a) No potential



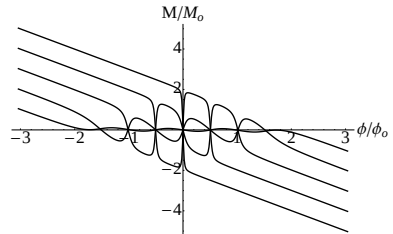
(b) Weak potential

Figure: Plot of the first five energy levels for a quantum ring using the free electron model (a) and using the free electron model with a weak potential (b).

Magnetic moment



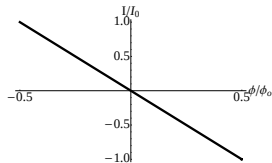
(a) No potential



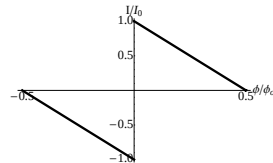
(b) Weak potential

Figure: Plot of the first five magnetic moment levels for a quantum ring using the free electron model (a) and using the free electron model with a weak potential (b).

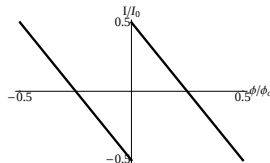
Persistent currents



(a) N_e odd



(b) N_e even



(c) Averaged over N_e

Figure: For N_e even and N_e odd the p.c. are different but both have the same periodicity. Averaging the persistent currents over N_e result in a halving of the period and of the maximum current.

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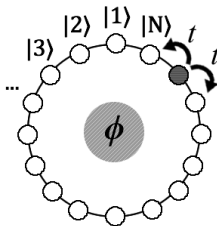
Description

- Independent electrons
- Electrons are strongly bound to ions
- Spinless fermions

Hamiltonian

Tight-binding Hamiltonian with flux

$$H = -t \sum_{j=1}^N \left(e^{i(\phi'/N)} c_j^\dagger c_{j+1} + h.c. \right) \quad (5)$$



Energy and magnetic moment

Energy

$$E = -2t \cos \left(k - \frac{\phi'}{N} \right) \quad (6)$$

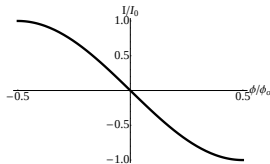
with k lying in the first Brillouin zone.

Magnetic moment

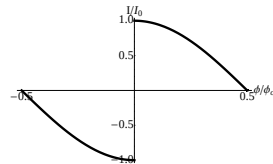
$$M = -M_o \sin \left(k - \frac{\phi'}{N} \right) \quad (7)$$

where $M_o = \frac{2t\pi R^2}{N\phi_o}$.

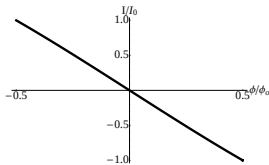
Persistent currents



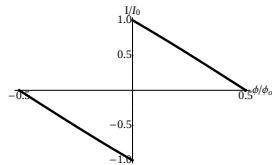
(a) $N = 2, N_e$ odd



(b) $N = 2, N_e$ even



(c) $N = 7, N_e$ odd



(d) $N = 7, N_e$ even

Figure: Persistent currents for an odd and an even number of electrons.

One impurity ring - Description

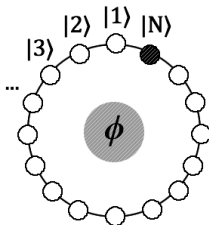


Figure: A ring enclosing an external flux with an impurity at site N .

Hamiltonian

Hamiltonian

$$\begin{aligned}
 H = & \varepsilon_0 \sum_{j=1}^N c_j^\dagger c_j - t \sum_{j=1}^{N-2} \left(e^{i(\phi'/N)} c_j^\dagger c_{j+1} + h.c. \right) \\
 & + \varepsilon_N c_N^\dagger c_N - t_N \sum_{j'=N-1}^N \left(e^{i(\phi'/N)} c_{j'}^\dagger c_{j'+1} + h.c. \right)
 \end{aligned} \tag{8}$$

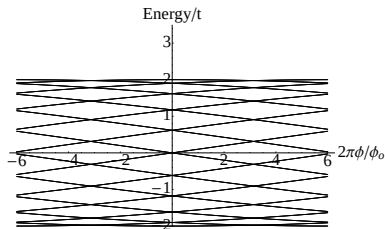
ε_N - impurity on-site energy

ε_0 - non impurity on-site energy

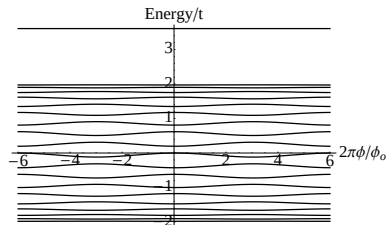
t_N - hopping factor from and to the impurity

t - the hopping factor between non-impurity sites

Bound state



(a) Perfect ring



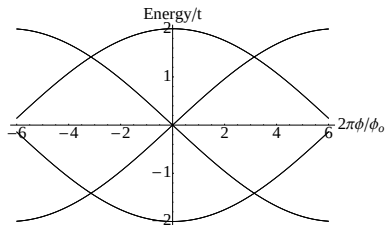
(b) Ring with impurity

Figure: Energy vs magnetic flux plot for a perfect ring and for a ring with an impurity with $\varepsilon_0 = 3$ ($N = 20$).

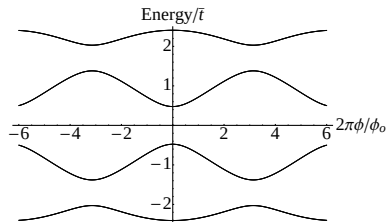
Disordered ring - Description

- In a disordered ring we can consider that t is different between different sites
- We can also consider that the on-site energy is different between different sites

Energy



(a) Perfect ring



(b) Disordered ring

Figure: Energy vs magnetic flux plot for a perfect ring and a disordered ring ($N = 4$ and t_j chosen randomly).

Ensemble persistent current

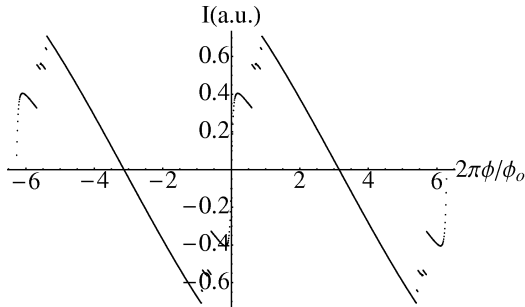


Figure: Persistent current curves for an *ensemble* of disordered rings (averaged over 100 disorder configurations) with t_j chosen randomly, $N = 4$ and $\varepsilon_F = -0.8$

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Description

t-V Hamiltonian with flux

$$H = -t \sum_{j=1}^N \left(c_j^\dagger c_{j+1} + h.c. \right) + V \sum_{j=1}^N (n_j n_{j+1}). \quad (9)$$

where V is the Coulomb energy between two nearest-neighbors electrons and $n_j = c_j^\dagger c_j$ is the occupation number at site j .

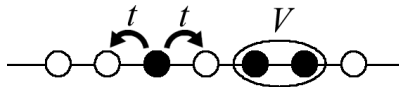
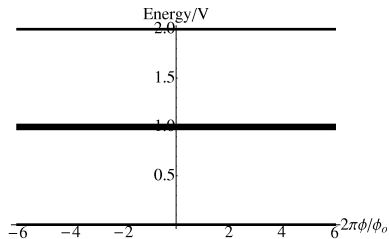
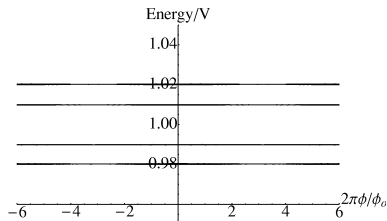


Figure: The t-V model is a spinless tb model with Coulomb interaction between nearest-neighbors.

Energy in the strong coupling limit ($V/t \gg 10$)



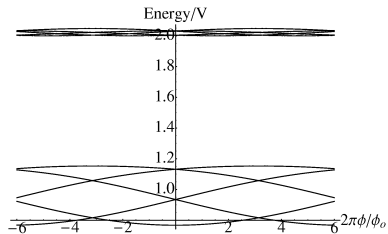
(a) Global view



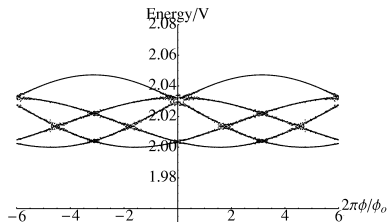
(b) Zoom in region $V=1$

Figure: Energy-flux plots for a system with $N = 6$ and $N_e = 3$ in the strong coupling limit.

Energy when not in the strong coupling limit



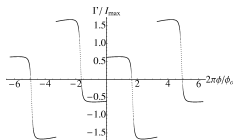
(a) Global view



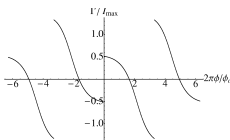
(b) Zoom in the top energy subspace

Figure: Energy plot for a ring with $N = 5$ and $N_e = 3$.

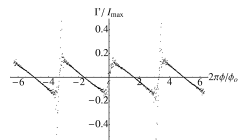
Persistent currents



(a) $t = 1$, $V = 0.1$,
period = 2π



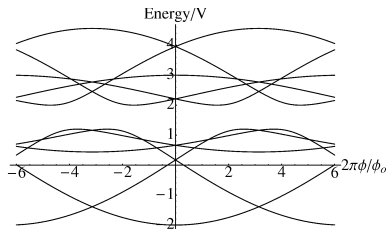
(b) $t = 1$, $V = 1$,
period = 2π



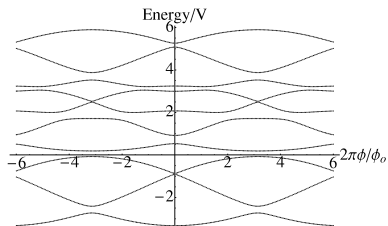
(c) $t = 1$, $V = 10$,
period = π

Figure: P.c. for the fundamental level of the top subspace for a system with $N = 5$ and $N_e = 3$.

One impurity t-V ring



(a) Without impurity



(b) With impurity

Figure: Comparison of the energy vs flux plots for a t-V ring with $N = 5$ and $N_e = 3$ with and without an impurity.

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- Quick convergence of the p.c. results for the free electron gas ring and the tb ring
- Averaging the p.c. over N_e results in a halving of the energy/p.c. periodicity
- In a tb ring an impurity is responsible for a bound state
- In a t-V ring some energy levels can have a decreased periodicity
- In the strong coupling limit the energy levels are localized around integer values of V
- In general, an impurity lifts the energy degeneracy as expected

Webpage

<http://www.quantumrings.vndv.com>